Wideband High-Selectivity Diplexers Utilizing Digital Elliptic Filters

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Abstract—Design techniques and element value tables are presented for the construction of compact high-selectivity diplexers using digital elliptic component filters. A brief review of an applicable diplexer theory is given and the derivation and use of the element value tables are discussed. The tables provide values for component filters with an even number of branches n for $n=4$ through $n=12$ with a prototype ripple value corresponding to maximum diplexer input VSWR of approximately 1.26 to 1. For $n=8$, 10, and 12 branch filters, additional tables with permuted transmission zero orders are given.

Test results for an $n=6$ branch experimental diplexer of 2.25 to 1 bandwidth are presented. The design and construction of this diplexer is described in detail and serves to illustrate the use of the element value tables and design procedures presented. Many practical construction and alignment suggestions are given which are useful in obtaining designs with good response characteristics. The experimental diplexer has crossover frequencies at 1.5 and 3.4 GHz and provides greater than a 50 dB isolation at frequencies 0.2 GHz from crossover. The diplexer has an input VSWR $\leq$1.5 to 1 from dc to 6.0 GHz, and has package dimensions of approximately 2.0 by 2.0 by 0.75 inches.

I. Introduction

A PROBLEM often encountered at microwave frequencies is the separation of contiguous frequency bands using passive filters. Detailed design techniques have been presented[1]–[4] for achieving suitable multiplexers; however, the structures used to realize the component filters often have been large and bulky, especially for high-selectivity wideband requirements. The high-selectivity of digital elliptic filters[5]–[7] makes them attractive for use in wideband diplexers, and their compact size results in structures that are substantially smaller than previous designs. This paper presents design techniques and element value tables that allow the simple design of compact diplexers and multiplexers using digital elliptic component filters. The tables presented give element values for filters of 4, 6, 8, 10, and 12 branches. These tables provide sufficient complexity to satisfy most wideband requirements.

This paper is presented in the following manner. In Section II, a brief review of diplexer filter theory relevant to digital elliptic filters is given. General criteria for multiplexer design are stated and related to elliptic function type responses. In Section III, design techniques and element value tables are presented for the design of wideband high-selectivity diplexers. Both the derivation and use of the element value tables are discussed. In Section IV, experimental results for a wideband diplexer model are presented. The design is described in detail and serves to illustrate the application of the tables to a typical diplexer requirement. Practical construction techniques and alignment suggestions are also presented. Conclusions and an overall evaluation of digital elliptic diplexers are given in Section V.

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II. Digital Elliptic Diplexing Filters

The theory of diplexers has been described in several papers[1]–[4] where it is shown that a perfect match at the input port of a diplexer requires the component filters to be complementary. Filters with equal-ripple response in both passband and stopband can be designed to be complementary; however, it has been pointed out[2] that this places an undesirable restriction on the isolation characteristics. The use of "pseudo-complementary filters"[1],[2] allows the achievement of equal-ripple designs with high-isolation characteristics at the cost of a slight increase in the input VSWR. The performance of such filters is described in detail in Wenzel[2] where it is shown that for proper operation the component filters should:

1) be designed on a singly terminated or transfer immittance basis ($|Z_{12}|^2$ for series connection and $|Y_{12}|^2$ for parallel connection),

2) have attenuation characteristics that "cross over" at the 3 dB level,

3) have component attenuation characteristics whose slopes are equal and of opposite sign at the crossover frequency, and

4) have a total real part input immitance that is approximately constant (to within 20 percent, for example) and devoid of extremely rapid variations.

Under these conditions, the maximum input VSWR (VSWR_M) is given to a good approximation by[2]

$$\text{VSWR}_M \approx 10^{\alpha/10} \tag{1}$$

where α is the transfer immittance prototype ripple value in decibels. Although the transfer immittance function can have relatively large ripples, the actual power transmission ripple is determined by the resulting input VSWR. For example, a 1 dB $|Y_{12}|^2$ design gives from (1) a VSWR_M

$=1.26$, and a corresponding power transmission ripple of approximately 0.06 dB.

In general, conditions 1) through 4) can be satisfied by: a) synthesizing a singly terminated lowpass prototype filter, b) bandwidth scaling this design to have the 3 dB frequency occur at the desired crossover frequency, and c) using a low to highpass transformation to achieve the complementary or pseudo-complementary highpass prototype.

Digital elliptic filter prototypes contain only distributed L-C type elements (i.e., no unit elements) and a suitable diplexer prototype is readily obtained by bandwidth scaling the appropriate lowpass transfer immittance values in the conventional manner[8],[9] to produce the required 3 dB level crossover. Element values for singly terminated elliptic function filters of three through seven branches (one to three finite transmission zeros) are given in Skwirzynski.[10] The particular filter configuration to be described uses a parallel connection of digital elliptic component filters and thus requires the input admittances to be minimum susceptive.[2],[8],[9] This, in turn, requires the lowpass prototype to begin with a series inductance and the highpass prototype to begin with a series capacitance at the junction of the two component filters. Consequently, the prototype filters are limited to those having an even number of branches (i.e., 4, 6, etc.).[1] Although filters of four or six branches are of sufficient complexity to satisfy many requirements, higher-order filters are sometimes needed. To satisfy high-selectivity requirements and to obtain a consistent set of tables applicable to microwave digital elliptic realizations, element values have been computed for prototypes with an even number of branches n for $n=4$ through $n=12$. The derivation and use of these tables is described in Section III.

III. Element Value Tables for Digital Elliptic Diplexers

A. Derivation of Element Value Tables

The element value tables were derived using the detailed theory and synthesis procedures described by Skwirzynski (in particular Chapters 2 through 6).[10] Because of the tedious nature and length of these calculations, only those aspects of general interest will be discussed. In essence, the utilization of elliptic functions allows the closed form factorization of the transfer admittance function. With suitable manipulation, these roots are used to generate an appropriate reactance function from which the desired network can be obtained using well-known synthesis techniques.[8]–[10] The use of elliptic functions to obtain closed form solutions is very important in obtaining accurate element values for complex filters. Using the parameter methods described in Skwirzynski, reactance functions were obtained whose coefficients were very accurately known without resorting to specialized transformation techniques.[11]

Initially, the lowpass prototype was synthesized with the prototype ripple value[2] occurring at $\Omega_C=1$. This lowpass prototype was then bandwidth scaled to obtain $\Omega_{3\text{ dB}}=1$. As a check on the accuracy of the element values and the maximum input VSWR, the response of each diplexer derived from the tables was computed by direct analysis. The uniformity of the resultant maximum VSWR for all tables is good indication that the element values are highly accurate. Those element values which could be checked against the tables in Skwirzynski[10] ($n=4$ and 6) showed agreement to five significant figures (the maximum given in Skwirzynski). The maximum VSWR was in close agreement with (1) for all cases computed.

An important practical aspect of the synthesis procedure is the physical order in which transmission zeros are produced. That is, for K finite transmission zeros, there are $K!$ possible physical networks, each possessing resonant circuits that realize the K transmission zeros in a different order. For example, with the $n=12$ branch filter, there are five (5) transmission zeros or $5!=120$ possible network realizations. Consequently, a decision had to be made as to the transmission zero order to utilize in the tables. Generally speaking, the order of transmission zero removal influences the number of negative element values that will be encountered as well as the numerical spread of the element values. For certain transmission zero ordering, negative element values occur for extreme values of passband ripple and selectivity. For other orders, the numerical spread of element values is larger than necessary. Previous work on doubly terminated filters has indicated that best results are achieved if transmission zeros closest to the cutoff frequency are realized physically in the center of the filter, and transmission zeros further from the band edge are realized physically at the ends of the filter.[12]–[14] To investigate these realization possibilities, element values for several permutations of the transmission zero locations were computed for each value of n, including choosing the transmission zeros in monotonically increasing order. The results of this investigation showed that both the monotonic order and the order that realized transmission zeros closest to cutoff in the center of the filter resulted in acceptable element values for singly terminated designs. Other permutations gave element values that were "different," but no improvement over the aforementioned orders was noted. In the tables to be presented, for $n=8$, 10, and 12, two transmission zero orders are given for each value of n. In these cases, the first table gives the monotonic order, and the second gives the permuted order with zeros closest to cutoff realized in the center of the filter. This last order is that utilized by Saal in his extensive tables of doubly terminated element values.[15]

[1] Elliptic function prototype filters with an odd number of branches can be utilized in diplexer design if a physical configuration and interconnection other than a parallel connection of digital elliptic component filters is considered.

[2] The prototype frequency variable is taken to be Ω, where $S=j\Omega=j\tan(\pi f/2f_0)$ is Richards' transformation.

B. Content of the Element Value Tables

The complete prototype and typical response characteristics for digital elliptic diplexers are shown in Fig. 1(a). The mapped response using Richards' transformation $S=j\Omega=j\tan(\pi f/2f_0)$ is shown in Fig. 1(b). The structure and response shown are for $n=12$ branches. The prototype for other values of n is identical in form where C_n and all other element values with subscripts greater than n are taken to be zero. For lower values of n there is also a corresponding reduction in the number of finite transmission zeros. The element values in the tables are normalized such that the crossover frequency occurs at $\Omega_{3\text{ dB}}=1$ (i.e., $f_{3\text{ dB}}/f_0=\frac{1}{2}$) resulting in a 3 to 1 bandwidth for an unscaled distributed realization.

Element values are given in Tables I through VIII for $n=4$ through 12 branches with a transfer admittance prototype ripple value of 1 dB. This ripple value results in a maximum diplexer input VSWR of 1.26 to 1 and a power transmission ripple value of 0.06 dB. For $n=8$, 10, and 12, Tables IV, VI, and VIII, respectively, having permuted transmission zero locations (permuted tables are designed with a P) are included. The eight tables presented are adequate in satisfying most practical requirements.

The following symbols are utilized in the tables:

- k = selectivity parameter; ratio of the cutoff frequency to the frequency at which the attenuation first reaches A_s in decibels. For high selectivity, the cutoff frequency is essentially the crossover frequency
- C_i (i odd) = shunt capacitor
- C_i (i even) = series bridging capacitor
- L_i (i even) = series bridging inductor
- $\Omega_i=1/\sqrt{L_iC_i}$ (i even) = finite transmission zero frequencies
- A_s = minimum stopband attenuation level in decibels
- VSWR_M = theoretical maximum input VSWR of diplexer
- Ω_{A_s} = frequency at which the attenuation first reaches A_s in decibels.

In all cases, the tables have been reduced in size by limiting selectivity and attenuation levels to those values that are likely to be encountered in a practical requirement. For small n, the selectivity is given in steps of 0.05, while for large n it is given in steps of 0.02.

It is interesting to note that the normalized element values in the tables are relatively close to unity for all values of n and for a wide range in selectivity factor. Some small values of series bridging capacitors do occur, but these are realized by a parallel coupling of lines and are usually convenient. The uniformity of element values indicates that digital elliptic diplexer designs are practical to construct.

C. Use of Element Value Tables

The practical design of digital elliptic diplexing filters is identical to that of doubly terminated digital elliptic filters and all previous design information given in Wenzel and Horton[6],[7] is applicable. The only difference between singly and doubly terminated designs is the numerical value of the elements. However, there are several aspects of diplexer design that are not relevant to a single filter design and these will be discussed.

In designing a digital elliptic diplexing filter, one must decide whether a single or double crossover filter is desired. This consideration determines the center frequency f_0 that must be chosen. Because of the repeating response of distributed quarterwave filters [see Fig. 1(b)], the design may be either pseudo highpass-lowpass, or bandpass-bandstop. If only one crossover is desired in the frequency band of interest, f_0 can be varied to obtain a design with most convenient element values. If a bandpass-bandstop combination is desired, f_0 is constrained to be $(f_{C1}'+f_{C2}')/2$, where f_{C1}' and f_{C2}' are the crossover frequencies. The advantages of using the repeating response of distributed filters in multiplexers is discussed in Wenzel.[2]

Another important consideration in distributed filter design is the effect of the tangent mapping function on selectivity. The effect of the tangent mapping is to increase selectivity of the distributed filter in comparison with that of the prototype filter, the amount increasing as the bandwidth is reduced. Thus, it is important to check the mapped response to insure that the filter has not been overdesigned. This check is automatically included in the design procedures to be presented.

Complete design data for digital elliptic diplexing filters is given in Table IX. For filters with two crossovers, the crossover frequencies are f_{C1}' and f_{C2}'. For a single crossover, f_{C2}' is arbitrary and is chosen to yield a desired f_0. Table IX gives expressions for computing the required selectivity factor k and converting the lowpass prototype element values into static capacitances and stub impedances for both bandpass and bandstop component filters. These static capacitances and stub impedances can be utilized directly with available design data[16],[17] to obtain dimensions. The applicable techniques and detailed design examples are given in Wenzel and Horton.[6],[7] Many of the above considerations are described in the design example of Section IV.

The practical limitations of digital elliptic diplexers are the same as those listed in Wenzel and Horton. Several designs have been worked out in detail and practical dimensions have resulted for cases in which the component filters are wideband. These trial designs have indicated that bandwidths in the range of 2 to 1 (octave) to 4 to 1 usually result in practical structures.

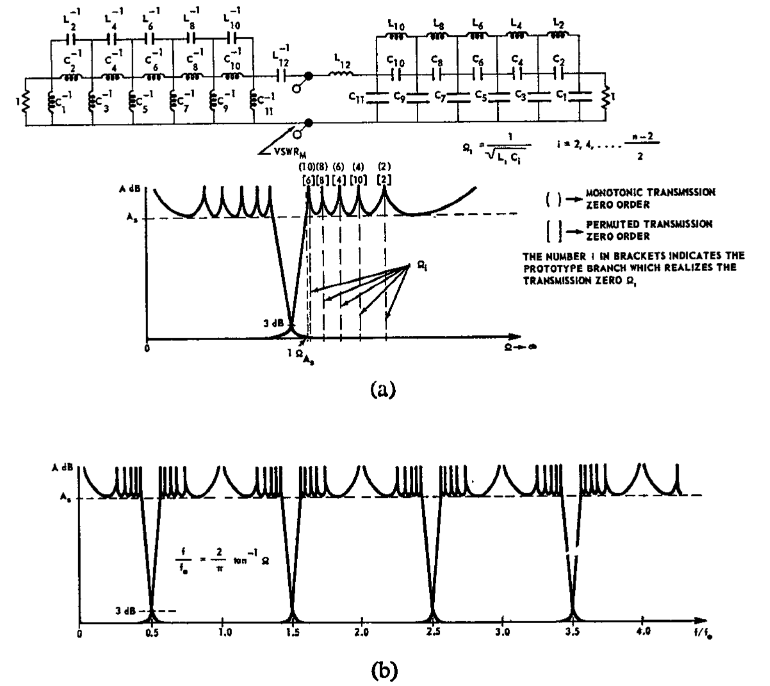


Fig. 1. Digital elliptic prototype and response characteristics. (a) L-C prototype and response characteristics. (b) Mapped response using Richards' transformation $S=j\Omega=j\tan(\pi f/2f_0)$.

TABLE I

Modified Lowpass Prototype Element Values for Normalized ($\Omega_{3\text{ dB}}=1$) Digital Elliptic Pseudo-Complementary Filter Pairs
$n=4$ Branches
Prototype Passband Ripple $=1.00$ dB
(0.06 dB Transmission Ripple)

k	C_1	C_2	L_2	C_3	L_4	Ω_2	A_s	$VSWR_M$	Ω_{A_s}
0.30	0.7749	0.0541	1.5562	1.6146	1.6606	3.4461	66	1.2645	3.1470
0.35	0.7533	0.0755	1.5185	1.6017	1.6619	2.9528	60	1.2647	2.6999
0.40	0.7276	0.1017	1.4740	1.5865	1.6634	2.5827	55	1.2648	2.3651
0.45	0.6974	0.1335	1.4223	1.5692	1.6651	2.2947	51	1.2649	2.1051
0.50	0.6623	0.1722	1.3627	1.5496	1.6670	2.0641	47	1.2650	1.8975
0.55	0.6215	0.2197	1.2945	1.5276	1.6691	1.8753	43	1.2650	1.7280
0.60	0.5741	0.2785	1.2167	1.5032	1.6714	1.7178	39	1.2652	1.5873
0.65	0.5188	0.3532	1.1279	1.4763	1.6736	1.5843	35	1.2651	1.4687
0.70	0.4536	0.4511	1.0264	1.4471	1.6757	1.4696	32	1.2646	1.3677
0.75	0.3755	0.5855	0.9100	1.4161	1.6771	1.3700	29	1.2634	1.2807
0.80	0.2793	0.7843	0.7753	1.3844	1.6766	1.2824	26	1.2599	1.2055
0.85	0.1551	1.1155	0.6178	1.3553	1.6716	1.2046	22	1.2527	1.1401

TABLE II

MODIFIED LOWPASS PROTOTYPE ELEMENT VALUES FOR NORMALIZED ($\Omega_{3\ \mathrm{dB}}=1$)
DIGITAL ELLIPTIC PSEUDO-COMPLEMENTARY FILTER PAIRS
$n=6$ BRANCHES
PROTOTYPE PASSBAND RIPPLE = 1.00 dB
(0.06 dB TRANSMISSION RIPPLE)

k	C_1	C_2	L_2	C_3	C_4	L_4	C_5	L_6	Ω_2	Ω_4	A_s	$VSWR_M$	Ω_{A_s}
0.30	0.8714	0.0283	1.5832	1.7021	0.0458	1.9167	1.6715	1.7093	4.7233	3.3736	112	1.2686	3.2553
0.35	0.8593	0.0393	1.5645	1.6779	0.0636	1.8803	1.6564	1.7097	4.0324	2.8912	104	1.2687	2.7914
0.40	0.8449	0.0526	1.5423	1.6494	0.0851	1.8375	1.6386	1.7101	3.5116	2.5292	96	1.2689	2.4437
0.45	0.8279	0.0685	1.5162	1.6162	0.1107	1.7878	1.6179	1.7106	3.1041	2.2477	90	1.2689	2.1734
0.50	0.8080	0.0874	1.4858	1.5781	0.1413	1.7306	1.5941	1.7111	2.7755	2.0223	83	1.2691	1.9573
0.55	0.7848	0.1099	1.4506	1.5344	0.1778	1.6652	1.5668	1.7117	2.5041	1.8379	77	1.2693	1.7808
0.60	0.7578	0.1370	1.4096	1.4846	0.2217	1.5907	1.5357	1.7124	2.2753	1.6841	72	1.2693	1.6338
0.65	0.7260	0.1699	1.3619	1.4278	0.2751	1.5056	1.5003	1.7131	2.0788	1.5539	67	1.2695	1.5097
0.70	0.6884	0.2105	1.3060	1.3626	0.3414	1.4084	1.4597	1.7138	1.9071	1.4422	61	1.2698	1.4035
0.75	0.6431	0.2621	1.2394	1.2876	0.4263	1.2962	1.4128	1.7146	1.7544	1.3452	56	1.2701	1.3117
0.80	0.5871	0.3304	1.1586	1.2000	0.5403	1.1653	1.3581	1.7153	1.6162	1.2602	51	1.2702	1.2318
0.85	0.5150	0.4272	1.0570	1.0957	0.7058	1.0090	1.2926	1.7159	1.4881	1.1850	46	1.2705	1.1616
0.90	0.4145	0.5828	0.9209	0.9670	0.9828	0.8142	1.2110	1.7159	1.3650	1.1179	41	1.2703	1.0998

TABLE III

MODIFIED LOWPASS PROTOTYPE ELEMENT VALUES FOR NORMALIZED ($\Omega_{3\ \mathrm{dB}}=1$)
DIGITAL ELLIPTIC PSEUDO-COMPLEMENTARY FILTER PAIRS
$n=8$ BRANCHES
PROTOTYPE PASSBAND RIPPLE = 1.00 dB
(0.06 dB TRANSMISSION RIPPLE)

k	C_1	C_2	L_2	C_3	C_4	L_4	C_5	C_6	L_6	C_7	L_8	Ω_2	Ω_4	Ω_6	A_s	$VSWR_M$	Ω_{A_s}
0.50	0.8844	0.0520	1.5259	1.6820	0.1013	1.7908	1.6062	0.1372	1.8013	1.5990	1.7286	3.5513	2.3480	2.0116	119	1.2694	1.9766
0.55	0.8695	0.0652	1.5053	1.6494	0.1270	1.7446	1.5565	0.1725	1.7344	1.5711	1.7289	3.1922	2.1246	1.8284	111	1.2697	1.7977
0.60	0.8520	0.0809	1.4812	1.6118	0.1576	1.6915	1.4996	0.2148	1.6580	1.5393	1.7291	2.8880	1.9371	1.6757	104	1.2693	1.6487
0.65	0.8313	0.0999	1.4527	1.5683	0.1943	1.6301	1.4346	0.2662	1.5710	1.5030	1.7293	2.6251	1.7769	1.5464	97	1.2699	1.5227
0.70	0.8066	0.1230	1.4188	1.5176	0.2391	1.5589	1.3598	0.3298	1.4715	1.4614	1.7295	2.3937	1.6380	1.4355	90	1.2701	1.4149
0.75	0.7767	0.1519	1.3778	1.4581	0.2931	1.4753	1.2733	0.4109	1.3568	1.4133	1.7297	2.1861	1.5156	1.3394	83	1.2696	1.3215
0.80	0.7393	0.1892	1.3269	1.3867	0.3677	1.3754	1.1717	0.5190	1.2230	1.3571	1.7298	1.9957	1.4063	1.2551	76	1.2704	1 2401
0.82	0.7214	0.2076	1.3027	1.3539	0.4034	1.3296	1.1258	0.5738	1.1628	1 3317	1.7299	1.9230	1.3654	1.2243	73	1.2700	1.2103
0.84	0.7014	0.2286	1.2757	1.3179	0.4445	1.2795	1.0761	0.6379	1.0979	1.3043	1.7298	1.8517	1.3260	1.1949	70	1.2691	1.1820
0.86	0.6787	0.2532	1.2451	1.2782	0.4926	1.2242	1.0219	0.7148	1.0275	1.2744	1.7298	1.7811	1.2877	1.1668	68	1.2707	1 1551
0.88	0.6525	0.2824	1.2099	1.2338	0.5501	1.1627	0.9623	0.8095	0.9505	1.2417	1.7297	1.7107	1.2504	1.1400	65	1.2708	1.1294
0.90	0.6216	0.3183	1.1686	1.1834	0.6210	1.0929	0.8961	0.9306	0.8653	1.2052	1.7295	1.6396	1.2138	1.1144	62	1.2680	1.1050
0.92	0.5838	0.3642	1.1187	1.1250	0.7124	1.0122	0.8210	1.0941	0.7695	1.1639	1.7291	1.5667	1.1776	1.0898	59	1 2710	1.0817
0.94	0.5355	0.4268	1.0556	1.0550	0.8386	0.9156	0.7336	1.3346	0.6591	1.1157	1.7284	1.4899	1.1412	1.0662	56	1.2691	1.0595
0.96	0.4683	0.5225	0.9691	0.9662	1 0351	0.7929	0.6271	1.7445	0.5264	1.0568	1.7272	1.4053	1.1038	1.0436	53	1.2705	1.0383
0.98	0.3553	0.7114	0.8289	0.8389	1.4367	0.6156	0.4840	2.7196	0.3523	0.9768	1.7242	1.3022	1.0633	1.0217	50	1.2592	1.0183

TABLE IV (P)

MODIFIED LOWPASS PROTOTYPE ELEMENT VALUES FOR NORMALIZED ($\Omega_{3\ \mathrm{dB}}=1$)
DIGITAL ELLIPTIC PSEUDO-COMPLEMENTARY FILTER PAIRS
$n=8$ BRANCHES, PERMUTED TRANSMISSION ZEROS
PROTOTYPE PASSBAND RIPPLE = 1.00 dB
(0.06 dB TRANSMISSION RIPPLE)

k	C_1	C_2	L_2	C_3	C_4	L_4	C_5	C_6	L_6	C_7	L_8	Ω_2	Ω_4	Ω_6	A_s	$VSWR_M$	Ω_{A_s}
0.50	0.8844	0.0520	1.5259	1.6444	0.1446	1.7090	1.6081	0.0963	1.8831	1.6346	1.7286	3.5513	2.0116	2.3480	119	1.2694	1.9766
0.55	0.8695	0.0652	1.5053	1.6036	0.1818	1.6450	1.5588	0.1208	1.8340	1.6146	1.7289	3.1922	1.8284	2.1246	111	1.2697	1.7977
0.60	0.8520	0.0809	1.4812	1.5568	0.2265	1.5721	1.5024	0.1499	1.7775	1.5914	1.7291	2.8880	1.6757	1.9371	104	1.2693	1.6487
0.65	0.8313	0.0999	1.4527	1.5032	0.2809	1.4888	1.4378	0.1850	1.7123	1.5648	1.7293	2.6251	1.5464	1.7769	97	1.2699	1.5227
0.70	0.8066	0.1230	1.4188	1.4414	0.3482	1.3936	1.3637	0.2277	1.6368	1.5338	1.7295	2.3937	1.4355	1.6380	90	1.2701	1.4149
0.75	0.7767	0.1519	1.3778	1.3695	0.4342	1.2838	1.2778	0.2812	1.5483	1.4974	1.7297	2.1861	1.3394	1.5156	83	1.2696	1.3215
0.80	0.7393	0.1892	1.3269	1.2847	0.5493	1.1556	1.1769	0.3505	1.4429	1.4540	1.7298	1.9957	1.2551	1.4063	76	1.2704	1.2401
0.82	0.7214	0.2076	1.3027	1.2461	0.6077	1.0978	1.1313	0.3846	1.3946	1.4340	1.7299	1.9230	1.2243	1.3654	73	1.2700	1.2103
0.84	0.7014	0.2286	1.2757	1.2041	0.6764	1.0355	1.0819	0.4238	1.3419	1.4122	1.7298	1.8517	1.1949	1.3260	70	1.2691	1.1820
0.86	0.6787	0.2532	1.2451	1.1582	0.7589	0.9679	1.0282	0.4697	1.2839	1.3881	1.7298	1.7811	1.1668	1.2877	68	1.2707	1.1551
0.88	0.6525	0.2824	1.2099	1.1075	0.8608	0.8938	0.9691	0.5245	1.2194	1.3613	1.7297	1.7107	1.1400	1.2504	65	1.2708	1.1294
0.90	0.6216	0.3183	1.1686	1.0505	0.9920	0.8118	0.9033	0.5920	1.1465	1.3309	1.7295	1.6396	1.1144	1.2138	62	1.2680	1.1050
0.92	0.5838	0 3642	1.1187	0.9854	1.1704	0.7194	0.8288	0.6788	1.0624	1.2956	1.7291	1.5667	1.0898	1.1776	59	1.2710	1.0817
0.94	0.5355	0.4268	1.0556	0.9086	1.4356	0.6127	0.7423	0.7982	0.9620	1.2534	1.7284	1.4899	1.0662	1.1412	56	1.2691	1.0595
0.96	0.4683	0.5225	0.9691	0.8131	1.8965	0.4842	0.6372	0.9828	0.8351	1.1999	1.7272	1.4053	1.0436	1.1038	53	1.2705	1.0383
0.98	0.3553	0.7114	0.8289	0.6800	3.0396	0.3152	0.4969	1.3550	0.6527	1.1228	1.7242	1.3022	1.0217	1.0633	50	1.2592	1.0183

TABLE V

Modified Lowpass Prototype Element Values for Normalized ($\Omega_{3\text{ dB}}=1$)
Digital Elliptic Pseudo-Complementary Filter Pairs
$n=10$ Branches
Prototype Passband Ripple = 1.00 dB
(0.06 dB Transmission Ripple)

k	C_1	C_2	L_2	C_3	C_4	L_4	C_5	C_6	L_6	C_7	C_8	L_8
0.80	0.8265	0.1234	1.4102	1.5244	0.2604	1.5135	1.3246	0.4060	1.3729	1.1418	0.5117	1.2444
0.82	0.8136	0.1351	1.3939	1.4978	0.2850	1.4783	1.2840	0.4459	1.3216	1.0942	0.5654	1.1835
0.84	0.7990	0.1485	1.3757	1.4684	0.3129	1.4393	1.2398	0.4918	1.2658	1.0427	0.6284	1.1179
0.86	0.7824	0.1640	1.3548	1.4355	0.3453	1.3958	1.1910	0.5457	1.2047	0.9867	0.7037	1.0467
0.88	0.7632	0.1823	1.3306	1.3983	0.3836	1.3466	1.1367	0.6103	1.1369	0.9253	0.7963	0.9687
0.90	0.7403	0.2046	1.3018	1.3554	0.4301	1.2899	1.0753	0.6905	1.0609	0.8571	0.9146	0.8825
0.92	0.7122	0.2328	1.2665	1.3047	0.4889	1.2229	1.0043	0.7944	0.9737	0.7801	1.0740	0.7856
0.94	0.6759	0.2707	1.2209	1.2422	0.5678	1.1405	0.9195	0.9388	0.8707	0.6907	1.3077	0.6739
0.96	0.6249	0.3271	1.1568	1.1600	0.6857	1.0321	0.8123	1.1659	0.7423	0.5823	1.7044	0.5396
0.98	0.5384	0.4332	1.0484	1.0346	0.9087	0.8671	0.6590	1.6357	0.5624	0.4374	2.6389	0.3634

k	C_9	L_{10}	Ω_2	Ω_4	Ω_6	Ω_8	A_s	$VSWR_M$	Ω_{A_s}
0.80	1.3548	1.7378	2.3975	1.5928	1.3394	1.2531	101	1.2702	1.2437
0.82	1.3292	1.7377	2.3044	1.5408	1.3027	1.2224	97	1.2699	1.2137
0.84	1.3016	1.7376	2.2127	1.4900	1.2674	1.1931	94	1.2679	1.1851
0.86	1.2715	1.7374	2.1216	1.4404	1.2334	1.1652	90	1.2631	1.1579
0.88	1.2385	1.7372	2.0302	1.3913	1.2005	1.1386	86	1.2658	1.1320
0.90	1.2017	1.7370	1.9375	1.3425	1.1684	1.1131	83	1.2701	1.1073
0.92	1.1601	1.7365	1.8415	1.2933	1.1371	1.0887	79	1.2701	1.0836
0.94	1.1115	1.7359	1.7395	1.2426	1.1061	1.0653	75	1.2598	1.0611
0.96	1.0520	1.7348	1.6256	1.1887	1.0750	1.0428	72	1.2598	1.0396
0.98	0.9708	1.7326	1.4838	1.1266	1.0426	1.0211	68	1.2631	1.0191

TABLE VI (P)

Modified Lowpass Prototype Element Values for Normalized ($\Omega_{3\text{ dB}}=1$)
Digital Elliptic Pseudo-Complementary Filter Pairs
$n=10$ Branches, Permuted Transmission Zeros
Prototype Passband Ripple = 1.00 dB
(0.06 dB Transmission Ripple)

k	C_1	C_2	L_2	C_3	C_4	L_4	C_5	C_6	L_6	C_7	C_8	L_8
0.80	0.8265	0.1234	1.4102	1.4070	0.4397	1.2678	1.1625	0.5173	1.2310	1.2526	0.2415	1.6321
0.82	0.8136	0.1351	1.3939	1.3731	0.4840	1.2175	1.1130	0.5706	1.1727	1.2113	0.2644	1.5932
0.84	0.7990	0.1485	1.3757	1.3360	0.5354	1.1628	1.0595	0.6328	1.1100	1.1664	0.2905	1.5503
0.86	0.7824	0.1640	1.3548	1.2951	0.5961	1.1027	1.0013	0.7068	1.0421	1.1172	0.3208	1.5024
0.88	0.7632	0.1823	1.3306	1.2494	0.6697	1.0362	0.9375	0.7971	0.9678	1.0627	0.3567	1.4484
0.90	0.7403	0.2046	1.3018	1.1975	0.7619	0.9614	0.8665	0.9114	0.8856	1.0014	0.4002	1.3863
0.92	0.7122	0.2328	1.2665	1.1372	0.8832	0.8757	0.7864	1.0636	0.7933	0.9311	0.4553	1.3132
0.94	0.6759	0.2707	1.2209	1.0646	1.0554	0.7745	0.6935	1.2830	0.6868	0.8479	0.5292	1.2237
0.96	0.6249	0.3271	1.1568	0.9719	1.3344	0.6485	0.5808	1.6459	0.5587	0.7440	0.6395	1.1067
0.98	0.5384	0.4332	1.0484	0.8365	1.9428	0.4736	0.4307	2.4624	0.3895	0.5979	0.8473	0.9299

k	C_9	L_{10}	Ω_2	Ω_4	Ω_6	Ω_8	A_s	$VSWR_M$	Ω_{A_s}
0.80	1.5235	1.7378	2.3975	1.3394	1.2531	1.5928	101	1.2702	1.2437
0.82	1.5078	1.7377	2.3044	1.3027	1.2224	1.5408	97	1.2699	1.2137
0 84	1.4905	1.7376	2.2127	1.2674	1.1931	1.4900	94	1.2679	1.1851
0.86	1.4712	1.7374	2.1216	1.2334	1.1652	1.4404	90	1.2631	1.1579
0.88	1.4493	1.7372	2.0302	1.2005	1.1386	1.3913	86	1.2658	1.1320
0.90	1.4242	1.7370	1.9375	1.1684	1.1131	1.3425	83	1.2701	1.1073
0.92	1.3944	1.7365	1.8415	1.1371	1.0887	1.2933	79	1.2701	1.0836
0.94	1.3579	1.7359	1.7395	1.1061	1.0653	1.2426	75	1.2598	1.0611
0.96	1.3099	1.7348	1.6256	1.0750	1.0428	1.1887	72	1.2598	1.0396
0.98	1.2366	1.7326	1.4838	1.0426	1.0211	1.1266	68	1.2631	1.0191

TABLE VII

MODIFIED LOWPASS PROTOTYPE ELEMENT VALUES FOR NORMALIZED ($\Omega_{3\text{ dB}}=1$)
DIGITAL ELLIPTIC PSEUDO-COMPLEMENTARY FILTER PAIRS
$n=12$ BRANCHES
PROTOTYPE PASSBAND RIPPLE = 1.00 dB
(0.06 dB TRANSMISSION RIPPLE)

k	C_1	C_2	L_2	C_3	C_4	L_4	C_5	C_6	L_6	C_7	C_8	L_8
0.80	0.8819	0.0869	1.4548	1.6231	0.1924	1.6038	1.4526	0.3175	1.4886	1.2607	0.4294	1.3600
0.82	0.8720	0.0951	1.4435	1.6015	0.2103	1.5768	1.4176	0.3477	1.4454	1.2167	0.4718	1.3060
0.84	0.8609	0.1045	1.4306	1.5773	0.2306	1.5468	1.3792	0.3823	1.3982	1.1689	0.5210	1.2469
0.86	0.8481	0.1153	1.4159	1.5502	0.2541	1.5129	1.3365	0.4225	1.3457	1.1164	0.5787	1.1828
0.88	0.8333	0.1280	1.3987	1.5192	0.2817	1.4741	1.2885	0.4702	1.2869	1.0582	0.6482	1.1119
0.90	0.8155	0.1435	1.3780	1.4831	0.3150	1.4289	1.2335	0.5285	1.2200	0.9927	0.7347	1.0325
0.92	0.7937	0.1629	1.3525	1.4398	0.3568	1.3745	1.1690	0.6027	1.1419	0.9176	0.8475	0.9421
0.94	0.7653	0.1889	1.3191	1.3856	0.4123	1.3063	1.0905	0.7032	1.0476	0.8288	1.0054	0.8359
0.96	0.7251	0.2272	1.2714	1.3126	0.4938	1.2140	0.9886	0.8556	0.9267	0.7179	1.2562	0.7049
0.98	0.6560	0.2979	1.1883	1.1971	0.6432	1.0674	0.8366	1.1505	0.7497	0.5627	1.7841	0.5241

k	C_9	C_{10}	L_{10}	C_{11}	L_{12}	Ω_2	Ω_4	Ω_6	Ω_8	Ω_{10}	A_s	$VSWR_M$	Ω_{A_s}
0.80	1.1217	0.5080	1.2556	1.3527	1.7426	2.8117	1.8002	1.4546	1.3086	1.2521	125	1.2710	1.2457
0.82	1.0733	0.5612	1.1944	1.3269	1.7426	2.6985	1.7366	1.4105	1.2739	1.2215	121	1.2699	1.2155
0.84	1.0211	0.6235	1.1282	1.2993	1.7424	2.5866	1.6743	1.3678	1.2407	1.1923	116	1.2691	1.1868
0.86	0.9644	0.6980	1.0565	1.2691	1.7422	2.4752	1.6129	1.3262	1.2088	1.1644	112	1.2638	1.1594
0.88	0.9023	0.7897	0.9781	1.2360	1.7420	2.3632	1.5519	1.2855	1.1780	1.1378	108	1.2600	1.1334
0.90	0.8335	0.9066	0.8913	1.1991	1.7417	2.2490	1.4905	1.2454	1.1482	1.1124	103	1.2600	1.1085
0.92	0.7558	1.0641	0.7937	1.1574	1.7413	2.1304	1.4280	1.2054	1.1192	1.0881	99	1.2600	1.0847
0.94	0.6661	1.2948	0.6812	1.1087	1.7407	2.0035	1.3627	1.1650	1.0908	1.0648	94	1.2700	1.0619
0.96	0.5575	1.6857	0.5460	1.0490	1.7397	1.8608	1.2916	1.1231	1.0627	1.0424	90	1.2628	1.0402
0.98	0.4134	2.6042	0.3685	0.9674	1.7377	1.6808	1.2069	1.0768	1.0341	1.0209	85	1.2599	1.0195

TABLE VIII (P)

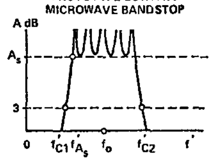
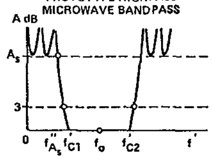
MODIFIED LOWPASS PROTOTYPE ELEMENT VALUES FOR NORMALIZED ($\Omega_{3\text{ dB}}=1$)
DIGITAL ELLIPTIC PSEUDO-COMPLEMENTARY FILTER PAIRS
$n=12$ BRANCHES, PERMUTED TRANSMISSION ZEROS
PROTOTYPE PASSBAND RIPPLE = 1.00 dB
(0.06 dB TRANSMISSION RIPPLE)

k	C_1	C_2	L_2	C_3	C_4	L_4	C_5	C_6	L_6	C_7	C_8	L_8
0.80	0.8819	0.0869	1.4548	1.5131	0.3429	1.3785	1.2354	0.5241	1.2171	1.1521	0.4328	1.3494
0.82	0.8720	0.0951	1.4435	1.4840	0.3762	1.3362	1.1877	0.5780	1.1596	1.1024	0.4758	1.2949
0.84	0.8609	0.1045	1.4306	1.4519	0.4144	1.2898	1.1361	0.6408	1.0978	1.0487	0.5258	1.2355
0.86	0.8481	0.1153	1.4159	1.4163	0.4591	1.2383	1.0797	0.7156	1.0306	0.9902	0.5845	1.1710
0.88	0.8333	0.1280	1.3987	1.3761	0.5126	1.1806	1.0175	0.8069	0.9572	0.9261	0.6552	1.0999
0.90	0.8155	0.1435	1.3780	1.3300	0.5784	1.1148	0.9479	0.9225	0.8759	0.8549	0.7434	1.0204
0.92	0.7937	0.1629	1.3525	1.2757	0.6630	1.0380	0.8687	1.0766	0.7846	0.7743	0.8584	0.9300
0.94	0.7653	0.1889	1.3191	1.2094	0.7794	0.9452	0.7759	1.2987	0.6791	0.6808	1.0194	0.8244
0.96	0.7251	0.2272	1.2714	1.1227	0.9597	0.8262	0.6615	1.6667	0.5522	0.5671	1.2746	0.6947
0.98	0.6560	0.2979	1.1883	0.9918	1.3226	0.6521	0.5047	2.4960	0.3844	0.4148	1.8062	0.5177

k	C_9	C_{10}	L_{10}	C_{11}	L_{12}	Ω_2	Ω_4	Ω_6	Ω_8	Ω_{10}	A_s	$VSWR_M$	Ω_{A_s}
0.80	1.3394	0.1750	1.7630	1.5709	1.7426	2.8117	1.4546	1.2521	1.3086	1.8002	125	1.2710	1.2457
0.82	1.3035	0.1915	1.7318	1.5584	1.7426	2.6985	1.4105	1.2215	1.2739	1.7366	121	1.2699	1.2155
0.84	1.2644	0.2102	1.6970	1.5448	1.7424	2.5866	1.3678	1.1923	1.2407	1.6743	116	1.2691	1.1868
0.86	1.2211	0.2319	1.6579	1.5293	1.7422	2.4752	1.3262	1.1644	1.2088	1.6129	112	1.2638	1.1594
0.88	1.1728	0.2574	1.6134	1.5117	1.7420	2.3632	1.2855	1.1378	1.1780	1.5519	108	1.2600	1.1334
0.90	1.1180	0.2882	1.5615	1.4911	1.7417	2.2490	1.2454	1.1124	1.1482	1.4905	103	1.2600	1.1085
0.92	1.0544	0.3270	1.4996	1.4665	1.7413	2.1304	1.2054	1.0881	1.1192	1.4280	99	1.2600	1.0847
0.94	0.9779	0.3787	1.4222	1.4357	1.7407	2.0035	1.1650	1.0648	1.0908	1.3627	94	1.2700	1.0619
0.96	0.8802	0.4547	1.3185	1.3941	1.7397	1.8608	1.1231	1.0424	1.0627	1.2916	90	1.2628	1.0402
0.98	0.7377	0.5942	1.1554	1.3282	1.7377	1.6808	1.0768	1.0209	1.0341	1.2069	85	1.2599	1.0195

TABLE IX

DESIGN EQUATIONS FOR DIGITAL ELLIPTIC DIPLEXERS

	PROTOTYPE LOWPASS MICROWAVE BANDSTOP	PROTOTYPE HIGHPASS MICROWAVE BANDPASS
		
BANDWIDTH SCALING, FACTOR, Ω_c'	$\Omega_c' = \tan \frac{\pi f'_{C1}}{2 f_o} = \tan \frac{\pi f'_{C1}}{f'_{C1} + f'_{C2}}$ (T-1) $\% \text{BW} = 200 \left(1 - \frac{2}{\pi} \tan^{-1} \Omega_c'\right)$ (T-2)	
SKIRT* SELECTIVITY FACTOR, k	$k_L = \frac{\Omega_c'}{\Omega'_{A_s}} = \frac{\Omega_c'}{\tan \frac{\pi f'_{A_s}}{2 f_o}}$ (T-3)	$k_H = \frac{\Omega''_{A_s}}{\Omega_c'} = \frac{\tan \frac{\pi f''_{A_s}}{2 f_o}}{\Omega_c'}$ (T-4)
PARALLEL CONDUCTOR ARRAY	$c_{\text{chart}} = \frac{376.7}{\sqrt{\epsilon_r} Z_o} \frac{C}{\Omega_c'}$ (T-5)	$c_{\text{chart}} = \frac{376.7}{\sqrt{\epsilon_r} Z_o} C \Omega_c'$ (T-6)
INTERNAL SERIES STUBS	$\sqrt{\epsilon_r} Z_{\text{ser, stub}} = \sqrt{\epsilon_r} Z_o \frac{L}{\Omega_c'}$ (T-7)	$\sqrt{\epsilon_r} Z_{\text{ser, stub}} = \sqrt{\epsilon_r} Z_o L \Omega_c'$ (T-8)

A_s – MINIMUM STOPBAND ATTENUATION LEVEL IN dB
f'_{A_s} – LOW PASS FILTER FREQUENCY FOR WHICH THE ATTENUATION IS A_s dB
f''_{A_s} – HIGH PASS FILTER FREQUENCY FOR WHICH THE ATTENUATION IS A_s dB
f'_{C1} – LOWER 3 dB CROSSOVER FREQUENCY
f'_{C2} – HIGHER 3 dB CROSSOVER FREQUENCY
$f_o = (f'_{C1} + f'_{C2})/2$ – FREQUENCY FOR WHICH THE LINES ARE QUARTER-WAVELENGTH
k – SKIRT SELECTIVITY PARAMETER LISTED IN ELEMENT VALUE TABLES
L, C – LOW PASS PROTOTYPE ELEMENT VALUES LISTED IN ELEMENT VALUE TABLES
Z_o – CHARACTERISTIC TERMINATING IMPEDANCE (ohms)
ϵ_r – RELATIVE DIELECTRIC CONSTANT OF MEDIUM
c_{chart} – "STATIC CAPACITANCE" TO BE FOUND IN CHARTS [16]
$\sqrt{\epsilon_r}\ Z_{\text{ser, stub}}$ – CHARACTERISTIC IMPEDANCE TO BE FOUND IN CHART [17]
*CHOOSE $k \geq k_H$ IF $\Omega_c' > 1$ AND $k \geq k_L$ IF $\Omega_c' < 1$

IV. EXPERIMENTAL RESULTS INCLUDING CONSTRUCTION AND ALIGNMENT SUGGESTIONS

To illustrate in detail the design of digital elliptic diplexers using the element value tables and design Table I, consider the following specifications typical of those likely to be encountered:

1) Bandpass-bandstop diplexer with crossover frequencies $f_{C1}' = 1.50$ GHz and $f_{C2}' = 3.375$ GHz (i.e., 2.25 to 1 bandwidth)
2) Isolation greater than 60 dB at $f_{C1}' \pm 0.30$ GHz (i.e., $f_{A_s}'' = 1.20$ GHz and $f_{A_s}' = 1.80$ GHz)
3) Maximum input VSWR≤ 1.5 to 1.

From design Table IX one determines

$$f_0 = \frac{f_{C1}' + f_{C2}'}{2} = \frac{4.875}{2} \approx 2.44 \text{ GHz}. \tag{2}$$

Then from equation (T-1)

$$\Omega_c' = \tan \frac{\pi f_{C1}'}{2 f_0} = \tan 55.4° = 1.45 \tag{3}$$

and from equation (T-4)[3]

$$k \geq k_H = \frac{\tan \dfrac{\pi f_{A_s}''}{2 f_0}}{\Omega_c'} = 0.672. \tag{4}$$

The desired parameters are now completely specified. That is, one wishes to find in the tables a design with $k \geq 0.672$, $A_s \geq 60$ dB, and $\text{VSWR}_M \leq 1.5$ to 1. The calculations of (2) through (4) determine the actual prototype selectivity required. A simple ratio of the highpass A_s in decibels attenuation frequency to the crossover frequency gives $k = 1.20/1.50 = 0.80$. However, as explained in Section III-C, the effect of the tangent mapping function is to increase the selectivity of the distributed realization over the prototype selectivity and the calculations of (2) through (4) indicate that any $k \geq 0.672$ is suitable. The use of the direct ratio $k = 0.80$ would lead to a filter that was overdesigned. If one wishes to insure that a specification will be met, some overdesign may be desirable.

Inspection of the element value tables shows that an $n = 6$ branch filter in Table II has the following satisfactory specifications:

$$k = 0.70, \qquad A_s = 61 \text{ dB},$$
$$\text{VSWR}_M \approx 1.27, \qquad \Omega_{A_s} = 1.404$$

frequencies of zero transmission

$$\Omega_2 = 1.907, \qquad \Omega_4 = 1.442. \tag{5}$$

The normalized low and highpass prototype element values are shown in Fig. 2(a). Bandwidth scaled real frequencies suitable for obtaining the actual frequency response are given by[6],[7]

$$f' = \frac{2 f_0}{\pi} \tan^{-1} \Omega_c' \Omega. \tag{6}$$

For the highpass prototype, one substitutes Ω^{-1} for Ω in (6) to obtain the bandwidth scaled frequencies. Applying (6) to the design data gives the final theoretical mapped response of Fig. 2(b).

Static capacitance values and series stub impedance values are obtained by substituting the normalized lowpass prototype values of Fig. 2(a) in equations (T-5) through (T-8) of Table IX. The static capacitance network for the complete diplexer along with stub impedance values is shown in Fig. 2(c). Dimensions for the coupled-line portions of each filter are determined using Getsinger's data[16] as described in Matthaei *et al.*,[4] and Wenzel and Horton,[6],[7] and the series stub dimensions are obtained from the graph in *The Microwave Engineer's Handbook and Buyer's Guide.*[17]

[3] *Note:* In determining the required selectivity factor, choose $k \geq k_H$ when $\Omega_c' > 1$ and $k \geq k_L$ when $\Omega_c' < 1$. The choice of k is influenced by the tangent mapping function. The values of k_H and k_L do not differ significantly for bandwidths in the 2 to 1 to 4 to 1 range.

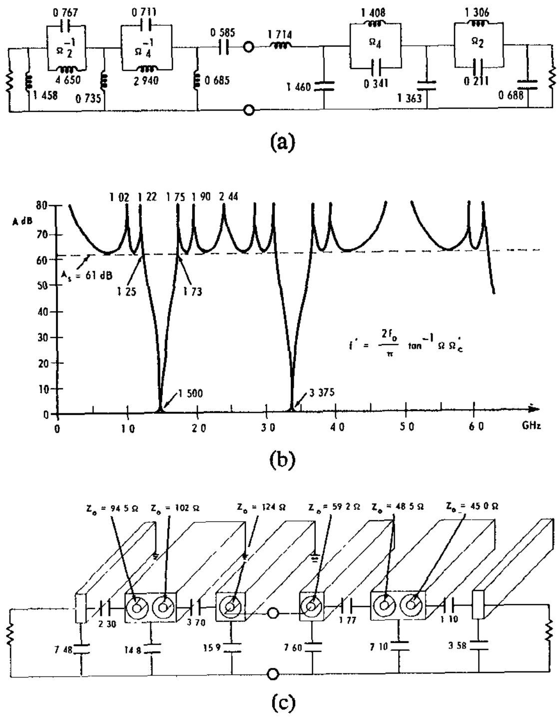


Fig. 2. Digital elliptic diplexer design example. (a) Diplexer prototype network obtained from element value Table II. (b) Mapped theoretical response characteristics. (c) Static capacitance network and series stub impedance values.

Inspection of the static capacitance values shows the shunt lines closest to the common junction to be quite wide. To obtain narrower width lines and to allow the side by side placement of the component filters, a wall was inserted between the component filters and the first shunt line of each filter was coupled to this wall. The widths of both first shunt lines were chosen to be $b/2$ (where b is the ground plane spacing) and dimensions were calculated using the procedures in Getsinger[16] to obtain the desired shunt static capacitance. Complete cross-sectional dimensions, as determined by the preceding procedures, and a plan view of the diplexer are given in Fig. 3. The ground plane spacing b was chosen to be 0.250 inch and the coupled-line thickness t was chosen to be 0.100 inch for the bandstop filter and 0.150 inch for the bandpass filter.

A perspective sketch of the complete diplexer showing the physical layout of the coupled lines is given in Fig. 4. In the design of the series stubs, the outer diameters were chosen such that the center conductors would have a standard diameter. Teflon loading was utilized for all stubs except the first series stub in the bandpass filter which was the highest impedance line (124 ohms). All coupled lines were initially 1.210 inches long ($\lambda/4$ at 2.44 GHz) with the teflon loaded lines being 1.210/1.44 inches = 0.850 inch long. The teflon loading of the series stubs was shortened by approximately 0.100 inch and the center conductors were extended to allow the inclusion of tuning slugs. The tuning slugs designated A in Fig. 4 were of teflon and provided tuning of the bandpass transmission zeros while the B slugs were of brass and provided tuning of the bandstop transmission zeros. In the initial design, the teflon tuning slugs were of sufficient length to extend beyond the back of the filter allowing adjustment with both ground planes intact. Likewise, the brass tuning slugs were threaded at the rear and nylon rods were inserted to provide external adjustment. The bandstop parallel-coupled lines were provided with small 0-80 set screws to firmly hold the brass shorting slugs in place after they were adjusted. The teflon tuning slugs were a tight fit in the shorting block and were cut off after final adjustment. It is important that the brass shorting slugs not extend beyond the end of the open-circuited parallel-coupled lines. In total, approximately 0.100 to 0.150 inch of adjustment in the lengths of the series stubs was obtained using the aforementioned procedures. This was found to be more than adequate in adjusting the filter. The slight air gaps that may occur due to these tuning adjustments were found to have no noticeable effects on the filter response. The tuning slugs are not necessary and could be eliminated in subsequent filters. They are, however, very convenient for obtaining the initial design dimensions.

In the model, most of the junctions were mitered to reduce discontinuities. Hardened berillium copper wire was used for the 0.020 inch center conductors and gold-plated tungsten wire was used for the 0.010 inch center conductors. The teflon cylinders with small center holes were realized using tandem sections one-half inch in length. The center conductor holes were made on a jeweler's lathe and little manufacturing difficulty was encountered. All coupled bars were machined from brass. The bandstop filter portion was supported by two $\frac{1}{32}$ inch machined fiberglass plates. A photograph of the complete model is shown in Fig. 5. The final dimensions of the entire structure are approximately 2 by 2 by 0.75 inch.

Before describing the final measured response characteristics, several practical alignment and construction techniques will be discussed. When the filter was initially tested, the attenuation response was fairly good, but the input VSWR was as large as 3 to 1 in the 2.0 to 3.0 GHz region. Adjustment of the tuning slugs further improved the attenuation characteristics but the input VSWR was lowered only slightly. Inspection of the structure (see Fig. 4) indicated that the junctions could be the cause of the high VSWR. The connecting lines between parallel lines and series stubs are assumed to be of zero length; however, at high frequencies this is unjustified. One must then answer the question as to what impedance a finite length of connecting line should be made when the design theory says it is of zero length. To answer this question, consider the bandpass filter in Fig. 4. In the passband, the series stubs are of low impedance and the shunt stubs are of high impedance. Therefore, between input and output ports there is a length of transmission line whose impedance is approximately the impedance of the connecting lines. Since these connecting lines were very small wires and consequently of high impedance, it was thought that increasing their size to achieve 50 ohm connecting lines would lower the VSWR. This was accomplished by increasing the size of the connecting lines and coupling them to the front wall as shown in Fig. 5. A brass block was added to achieve suitable coupling to the wall. Similar statements apply to the bandstop filter and the size of the connecting lines was increased accordingly. The results were quite gratifying in that the input VSWR was immediately reduced to

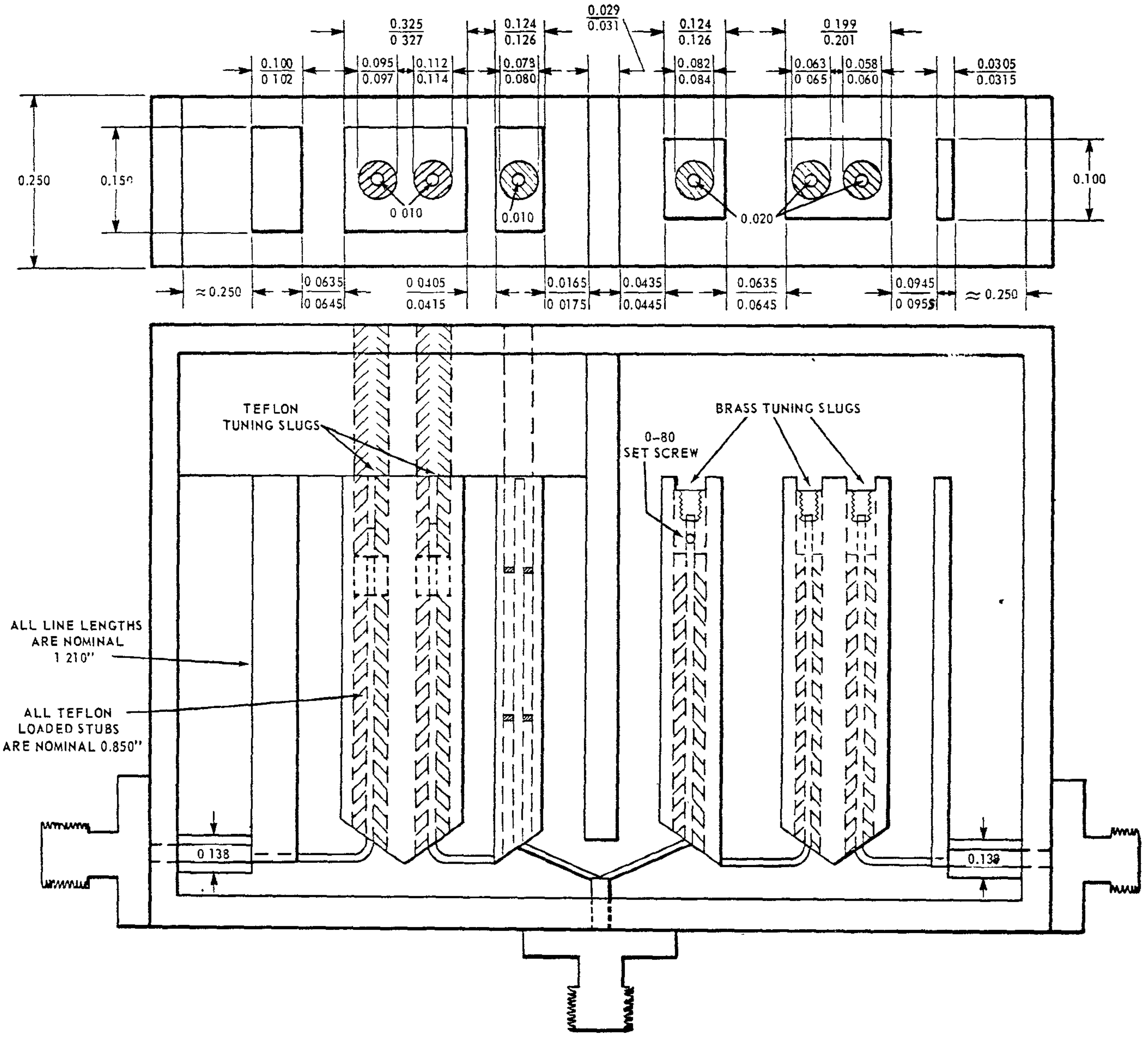


Fig. 3. Cross-sectional dimensions and plan view of experimental digital elliptic diplexer.

below 1.8 to 1 everywhere and below 1.5 to 1 over most of the dc to 6 GHz frequency band. It is suggested that for high-frequency filters the short connecting lines be made relatively large and the filter structures be placed sufficiently close to the front wall to achieve a near 50 ohm cross section. Filters for frequencies below 2.0 GHz should provide satisfactory performance without increasing the connecting line size.

In the filter design, the parallel-coupled lines are chosen to be one-quarter wavelength at the center frequency of the filter. Due to junction effects and finite widths, the exact lengths required cannot be determined analytically. Fortunately, good estimates of the lengths can be obtained by measurement. For example, consider the bandstop filter of Fig. 4 with the series shorting slugs removed. The series inductors are now capacitors and are practically shorts near $f=f_0$. One is then left with essentially the parallel capacitance array of Fig. 2(c). By shorting the ends of all but one of the coupled lines and by monitoring the insertion loss, a good estimate of the effective length of each conductor can be obtained. This length can then be adjusted to produce a transmission zero at $f=f_0$. A similar procedure can be utilized for the series stubs and also for all elements in the bandpass filter to obtain line lengths that are very close to those desired. In the experimental model, the line lengths in the bandstop filter were originally shortened to account for fringing effects. A measurement of the type described above then indicated that the center line was short and a tuning block was added as shown in Fig. 5 to achieve the desired effective length.

The measured response of the completed diplexer is shown in Fig. 6 and was obtained by adjusting all series

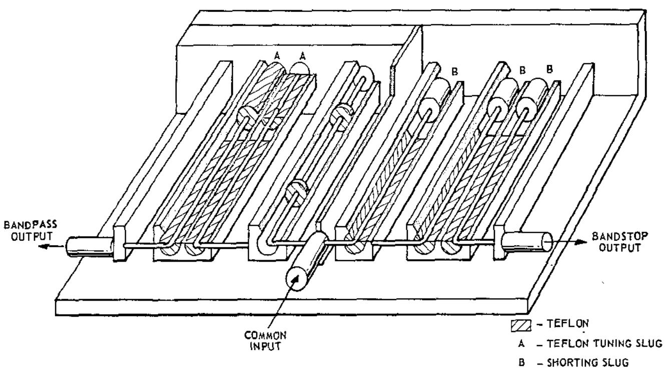


Fig. 4. Perspective sketch of experimental wideband digital elliptic diplexer.

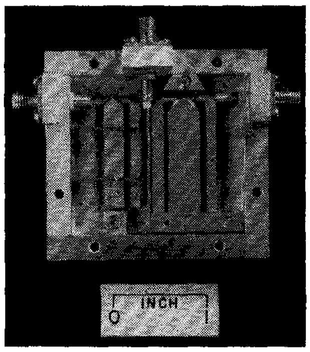


Fig. 5. Compact broadband digital elliptic diplexer.

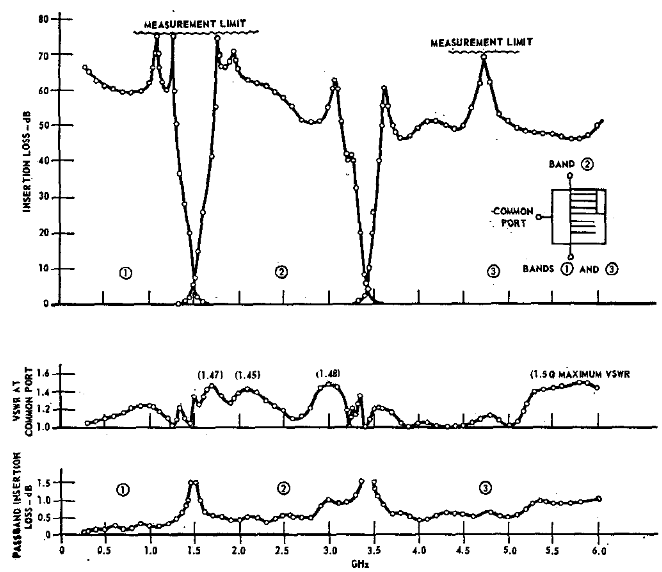


Fig. 6. Measured response of experimental digital elliptic diplexer.

shorting slugs after the connecting line sizes had been increased. This adjustment was carried out while monitoring input reflection only. The maximum input VSWR between dc and 6 GHz was 1.5 to 1. Inspection of the attenuation curves shows the lower crossover to be almost perfect, even exceeding the specifications at the high-frequency end of the crossover. The price paid for this increased attenuation is the lower than theoretical attenuation near the second crossover. The departure of the attenuation from theoretical is caused by slight misplacement of the bandstop transmission zeros as is evident in Fig. 6. There is also a tendency for dissipation loss and junction effects to obscure transmission zeros for higher repeating passbands. If the filter had been adjusted while monitoring the bandstop attenuation, some improvement could have been obtained, but the response was judged to be satisfactory as shown. A trial adjustment of transmission zeros using a one-watt amplifier and an oscilloscope enabled the setting of transmission zeros quite accurately. In performing an adjustment and measurement of this type, it is very important that the source be well filtered to eliminate harmonic outputs.

The measured insertion loss at both crossover frequencies was between 5.0 and 5.5 dB giving a dissipation loss of 2.0 to 2.5 dB. Passband loss was substantially lower as shown in Fig. 6. Since the filter had been handled quite extensively and no parts in the final structure were cleaned or plated, it is felt that the insertion loss could have been lowered. However, for the high selectivity and compactness of the structure, the measured values are quite encouraging. The crossover frequencies were very close to those predicted and the attenuation was good, especially at the first crossover.

Although the total multiplexer is a complex structure, the construction of the device was judged to be straightforward and no difficult problems were encountered.

V. Conclusions

Element value tables and design techniques for the construction of compact high-selectivity digital elliptic diplexers have been presented. The element values in the tables are very uniform and should result in practical filter structures particularly for bandwidths in the 2 to 1 to 4 to 1 range. Because the digital elliptic prototype is an L–C-type network, bandwidth scaling is simply accomplished and a large number of designs can be obtained from relatively few tables. The computed tables present element values heretofore unavailable, and these values are directly applicable to the design of lumped element diplexers as well as distributed digital elliptic realizations. The small size of the component filters will make them particularly desirable at UHF and low microwave frequencies; however, they should also find wide application at frequencies through *C* band.

By utilizing the repeating response of the distributed component filters, further size reduction in multichannel multiplexers can also be obtained. For example, the experimental diplexer described in Section IV can be made into a triplexer with the addition of a very simple and compact diplexer. This is possible because of the wide frequency separation between bands 1 and 3 shown in Fig. 6.

The test results for the experimental diplexer agreed well with theory and verified the design techniques presented. Although a multisection digital elliptic diplexer is a complex structure, little difficulty was encountered in constructing and aligning the experimental filter. The final experimental filter provided a combination of electrical and physical characteristics substantially better than those obtainable using other available techniques.

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Radial Line Band Rejection Filters in Coaxial Waveguides

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Abstract—**A coaxial waveguide with a cylindrical cavity forming a double discontinuity in the outer conductor is known to serve as a band rejection filter in the microwave region. A variational principle is applied to calculate the rejection frequency and a subsequent analysis is conducted to determine the dependence of that frequency on various parameters of the structure. Results are presented graphically and by simple analytical formulas. They demonstrate a newly discovered relationship between the rejection frequency and the width of the cavity, and provide design information which enables prediction of the rejection frequency within a 1 percent accuracy.**

Introduction

Among the simplest and least expensive structures that serve as band rejection filters in the microwave region is the coaxial waveguide with a cylindrical cavity forming a discontinuity in the outer conductor (Fig. 1). The band rejection properties of such structures are exploited in multiple frequency circuits, such as parametric amplifiers,[1] where frequency separation has very stringent requirements. When the outer conductor of a coaxial waveguide is perturbed to form a cylindrical cavity, the TEM mode is totally reflected at a resonant frequency that depends on as many as six parameters. These are the inner and outer radii of the coaxial line, the radius and width of the cavity, and the dielectric constants of the cavity and the line. The rejection frequency is more sensitive to some parameters than to others. Experience indicates that in restricted regions certain approximate methods, in which the effects of one or several of the less sensitive parameters are neglected, provide remarkably accurate results. However, there are discrepancies of 5 percent or more in other regions where the same approximations ought to be valid.[1],[2] The approximations most frequently used by filter designers correspond to either one of the following situations: a) total disregard of the fringing fields caused by the two close discontinuities in which case the cylindrical cavity is viewed as a series impedance equal to the input impedance of a shorted radial transmission line;[3] or b) consideration of the fringing fields associated with each discontinuity but neglect of the interaction between the two. In the latter, the discontinuities are accounted for by equivalent shunt-lumped reactive elements; however, they must be far enough apart so that the interaction is indeed negligible. A common feature of both cases is that they neglect to consider the cavity width.

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